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Numerical models seek to represent the interaction between landscape forms and processes through mathematical equations. By integrating these equations over space and time, numerical models have allowed geomorphologists to extend enquiry beyond observation alone, and explore landscape dynamics over a range of temporal and spatial scales. Choosing the correct temporal and spatial scale of investigation, the correct processes that control landscape form at these scales, and then converting this conceptual model to a mathematical representation of these process-form interactions is not straightforward. The decision requires careful consideration of process dominance and scale, the ability of equations to parameterise these processes, computational resources, and data availability to constrain model parameters and evaluate model performance. These issues shall be considered in general terms, and illustrated mainly with reference to catchment systems. Finally, numerical modelling of geomorphic systems is considered from a Bayesian perspective to provide a conceptual grounding for the development and application of numerical models, and therefore for their role in geomorphic enquiry.

KEYWORDS: Numerical modelling; Scale; Resolution; Evaluation; Data; Uncertainty.

Introduction

The interaction of landscape form and process may be represented mathematically in the form of a numerical model. Coupling such models with observations provides a formal framework to assemble scientific understanding, and a powerful tool to investigate landscape change. Numerical models provide some liberation from the temporal and spatial shackles imposed by enquiry through observation alone; models have allowed exploration of landscape processes and evolution over spatial scales ranging from particles to plate tectonics and temporal scales ranging from milliseconds to millennia (Figure 1; Bishop, 2007; Hardy, 2005). Numerical models have shown potential as powerful tools for understanding reductionist process-form interactions (e.g. Schmeeckle and Nelson, 2003; Wainwright et al., 2008a), and also the relative importance of autogenic versus allogenic controls for larger scale system behaviour (Coulthard et al., 2005; Nicholas and Quine, 2007b; Wainwright and Parsons, 2002).

Figure 1. Contrasting scales of model application: grain scale predictions of particle paths (black lines) over water worked gravel (modified from Hardy, 2005); convergent orogen formation modelling, considering techtonic uplift and surface erosion (modified from Willett and Brandon, 2002).
In order to fully exploit the potential for numerical models to elicit understanding of form-process interactions, and inform landscape management, numerical models must be developed and applied carefully considering three key questions:

1. What are the relevant form-process interactions at the scale of enquiry?
2. What are the correct mathematical representations of these processes?
3. Are there appropriate data to constrain model parameters and evaluate model predictions?

The main modelling issues that need to be considered in order to address these questions are presented here to provide a basis for more domain specific sections in Chapter 5, and illustrated mainly with reference to catchment systems. The strength of the assumptions made in developing and applying a model will determine the validity of model predictions, the strength of conclusions derived from model application, and therefore the ability of models to inform us of real world process-form phenomena. A Bayesian approach emphasising iterative dialogue between model development and data collection is recommended as a robust means to appropriately develop numerical models, and therefore geomorphic understanding.

Publications reviewing specific areas of geomorphic modelling (e.g. Bishop, 2007; Coulthard, 2001; Livingstone et al., 2007; Merritt et al., 2003; Morgan and Nearing, 2010; Pelletier, 2011; Reinhardt et al., 2010; Tucker and Hancock, 2010; Van de Wiel et al., 2011; Wainwright et al., 2008a), and publications expanding on the more general issues of model application to natural systems considered here (e.g. Beven, 2002; Bloschl and Sivapalan, 1995; Brazier et al., 2011; Church, 1996; Krueger et al., 2009; Nicholas, 2005; Refsgaard et al., 2006; Van de Wiel et al., 2011; Wainwright and Mulligan, 2004; Wilcock and Iverson, 2003) are additionally recommended.

Model structure

A model \( M \) contains equations with associated parameters \( \theta \) that represent the functional relationship between a vector of driving conditions \( D \) (e.g. rainfall), a vector of initial system states \( X_0 \) (e.g. landscape elevation), and vectors (with length \( t \), the length of the simulation) representing future system states \( X \), and outputs \( Y \) (e.g. catchment runoff/ sediment flux):

\[
Y, X = M(\theta, X_0, D)
\]  

Geomorphic models are generally concerned with the action of a number of processes which locally transport mass (e.g. sediment, including organics and nutrients) that lead to changes in landscape form \( (X) \) at a specific point over time \( (t) \). Though models generally differ in the processes evoked to move sediment, a fundamental approach governing most geomorphic models is to divide the landscape into units called control volumes (in 1, 2 or 3 dimensions). In most models a quasi-2D conservation of mass is applied by calculating the change in elevation \( d\eta \) in response to sediment flux into and out of a control, where \( dx \) indicates the size of the control in one dimension. The control may be divided into three stores, and \( M \) partly specified by (Figure 2; Tucker and Hancock, 2010; Wainwright et al., 2008a):

\[
\frac{\partial H_r}{\partial t} = T_u - S_c
\]

\[
\frac{\partial H_s}{\partial t} = d - \varepsilon + S_c
\]

\[
\frac{\partial H_t}{\partial t} = -\frac{\partial q_s}{\partial x} + \varepsilon - d
\]

where \( H_r \) is depth of bedrock (m), \( T_u \) is tectonic uplift (m); \( S_c \) (ms\(^{-1}\)) represents the rate of conversion of rock to soil/surface regolith, \( H_s \) (m); \( d \) is sediment deposition rate (ms\(^{-1}\)) and \( \varepsilon \) is sediment entrainment rate (ms\(^{-1}\)) from and into the equivalent depth of sediment in transport \( H_t \) (m), and \( q_s \) (m\(^2\)s\(^{-1}\)) is sediment discharge across the surface, \( dx \). Up to specifying the source terms, accounting for density/particle size differences between the stores, and developing an appropriate numerical solution, Equations 2-4 can generally be used to simulate the evolution of any point in the landscape, though some specific exceptions apply (Tucker and Hancock, 2010).
To completely specify M, first a conceptual understanding of the relevant source terms on the right hand side of equations 2-4 and the processes that control them are required at the scale of enquiry. Second, the conceptual model needs to be codified into a set of equations, and an appropriate analytical/numerical solution sought. As with all geomorphic enquiry, an understanding of scale underpins the specification of these source terms, and the answers to the questions posed in the introduction.

**Process dominance and scale**

The processes that govern changes in \( H_r \) operate over larger timescales (e.g. are relatively slower) than the processes governing changes in \( H_s \), which in turn are slower than processes governing changes in \( H_t \). Therefore as the timescale over which a model needs to be applied tends to zero, so the number of relevant stores and processes that control \( d\eta \) also reduce (Figure 2).

In catchment systems over millennia climate-induced fluctuations in sediment transport and rock breakdown, alongside tectonic uplift, govern the evolution of plate tectonics. Therefore, changes in \( H_r \) are important and need to be included in the conceptual model of landscape change (Bishop, 2007). Over decadal and centennial timescales, catchments predominantly evolve in response to climate-induced fluctuations in sediment transport, and therefore the stores of sediment in the landscape evolve through transport between control volumes, and \( H_r \) may be considered fixed and the processes that control them (\( T_u \) and \( S_c \)) relaxed. At this scale the state vector (X) needs to consider not only the evolution of sediment and mass, but also controlling and interacting factors such as vegetation (Istanbulluoglu and Bras, 2005; Reinhardt et al., 2010) and potentially anthropogenic influence (Wainwright and Millington, 2010). Alongside exerting control on sediment flux directly, these factors will also respond independently to climatic changes, creating potentially complex, and non-linear landscape feedbacks (Corenblit and Steiger, 2009). At even smaller temporal scales, sediment flux is controlled by current weather conditions and the effect of previous events operating at all scales (Schumm and Lichty, 1965), which manifest their effects through the model initial conditions. Therefore many system states are fixed (e.g. \( H_r \) and vegetation cover) and only need to be specified in the initial conditions (\( X_0 \)), with no additional equations required to simulate their evolution.

Similarly, over different spatial scales different processes will become important in controlling landscape behaviour. At small spatial scales on hill slopes instantaneous fluxes of water (raindrops and overland flow) control grain scale movements of sediment (Brazier et al., 2011). As catchment size increases overland flow concentrates to form rills, gullies and channels which, alongside mass movements, are increasingly important in controlling catchment sediment flux (Nichols, 2006). Therefore as the spatial and temporal scales of interest reduce, the range of processes that must be considered also reduces, and other, larger scale processes are manifest through the model boundary conditions (\( X_0 \)).

Developing a model to address a specific problem therefore requires a sound...
conceptual understanding of the time and space scales over which processes operate to control the landforms in question. The extent to which a process is represented in a model, however, depends on its mathematical formulation.

**Process representation**

Experimental work has been conducted, both in the laboratory and in the field, to investigate surface process, such as rain-splash and overland flow driven erosion (Furbish et al., 2009; Wainwright et al., 2000). Alongside fundamental, physical equations (e.g. Navier-Stokes equations of fluid motion), many such studies have provided us with predictive equations from which process-form interactions can be simulated (Tucker and Hancock, 2010; Wilcock and Crowe, 2003). When combined with conservation of mass equations and integrated over time, such models represent the fundamental mechanisms by which climatic fluctuations manifest in catchment-scale landscape evolution.

Given the temporal and spatial constraints on observation, the majority of experimental work has attempted to parameterise the processes in equation 4, through what may be determined process-based models (Wainwright et al., 2008a; Wilcock and Crowe, 2003). However, even at small spatial and temporal (reductionist/observational) scales, different process parameterisations have been developed.

In catchment systems erosion, transport and deposition of sediment by water is controlled by both transport limited processes ($T_L$; e.g. presence and power/stress imparted by water at the surface) and supply/detachment limited processes ($S_L$; e.g. the resistive forces at the sediment bed that impede sediment movement). The most widely applied predictive equations have calculated a sediment transport rate ($q_s$) as a function of $T_L$ and/or $S_L$, which implicitly assume sediment transport is in equilibrium (and potentially at some capacity), and evolve $H_S$ according to sediment flux into and out of the cell support (Wainwright et al., 2008a; Wilcock and Crowe, 2003). Though such methods may provide useful predictions, changes in sediment transport and soil depth are inherently in disequilibrium. Alternative parameterisations have been developed that explicitly calculate an entrainment and deposition flux into, and out of transport $H_t$ (Figure 3; Hairsine and Rose, 1992; Wainwright et al., 2008a). Developing better parameterisations of equation 4 have been limited by the difficulty of measuring sediment in transport. Therefore, even at scales where monitoring can take place to parameterise system processes, competing process representations may be derived reflecting process uncertainty, and also different experimental setups (Wainwright et al., 2000).

![Figure 3. Spatial pattern of at-a-point total sediment movement (kg) predicted by MAHLERAN soil erosion model when applied to an 18x35m runoff plot (Wainwright et al.2008b).](image-url)
sediment interactions should be modelled using these equations (Hardy, 2008), various simplifications are typically made because of computational limitations involved in deriving accurate numerical solutions – a class of models often termed “reduced complexity” models.

In the case of hydraulic/hydrologic modelling, such simplifications include depth averaging to two dimensions (Lane et al., 1999) and the fully dynamic 1D St Venant equations with further simplifications thereof that neglect potentially unimportant terms to derive the diffusive- and kinematic-wave models (Hunter et al., 2007; Tucker and Hancock, 2010). Additional flow simplifications have led to a number of cellular approaches for flow routing, which employ simplified rules to route flow in river channels and hillslopes (Favis-Mortlock, 1998; Nicholas, 2009; Thomas and Nicholas, 2002). Furthermore, some approaches have sought to employ simpler cellular and physically based rules to simulate morphological change, and have simulated sand dune formation (Figure 4; Nield and Baas, 2008), meander migration (Coulthard and Van De Wiel, 2006), braided river evolution (Thomas et al., 2007), and floodplain evolution (Figure 5; Karssenberg and Bridge, 2008).

The specific conditions under which such equations are ‘valid’ should be carefully considered (see Lane (1998) and Cao and Carling (2002) for a consideration of hydraulic equations in a geomorphic context). The scale and method by which transport limited conditions are modelled may be more important than the equations that link properties of flow to actual sediment movement, given the potential for non-linear error propagation.

The increased availability of high resolution topographic data has facilitated the application of small scale model parameterisations over increasingly large domains. For computational reasons and availability of other distributed data, however, it is often necessary to model at coarser resolutions in both space and time. One approach to deal with the problem of process parameterisation at coarser scales is simply to apply the same equations developed at smaller scales. However, a key problem with this approach is that many if not all geomorphic laws are scale dependent. For example many geomorphic laws are slope dependent, thus increasing model cell size reduces slope, and can lead to inaccurate predictions of erosion (Kalin et al., 2003). Such inaccuracies occur because of how changing scale of resolution affects both geomorphic laws and the laws governing flow (Brazier et al., 2011). Inferring the validity of a model equation independently of the grid within which it is applied may be difficult (Nicholas, 2005). Thus in model development changing model scale will affect the validity of all processes included in the model.

Although all spatial parameterisation is lumped to some degree, critical scales in the landscape that govern larger scale behaviour (e.g. the scale of interest) should be considered. For example, on hillslopes and in channels coarser scale models will fail to account for the spatial heterogeneity of flow. Given the relationship between sediment transport and flow is strong and non-linear, neglecting this heterogeneity will lead to an under-prediction of erosion and sediment transport (Ferguson, 2003; Nicholas, 2000). Relying on equilibrium concepts to model sub-grid scale channel features is a popular approach to deal with this problem in landscape evolution models, but relies on equilibrium concepts to model potentially non-equilibrium behaviour (Nicholas and Quine, 2007a; Tucker and Hancock, 2010). Therefore the specific scale at which a model equation is a valid representation of sub grid-scale processes is an important consideration when developing a numerical model.

At larger temporal scales of enquiry models have to deal with the disparity between the timescales of individual events (e.g. rainfall runoff) and the evolution of landscapes...
(Tucker and Hancock, 2010). As a result of computational limitations, many landscape evolution models applied over larger temporal scales relate at-a-point discharge to upslope contributing area, which implies runoff is in equilibrium with uniform rainfall. Such approximations subsequently used to simulate sediment transport fail to account for spatial and temporal variability. Sub-scale events may be important in controlling runoff production and therefore the sediment transport that actually governs landscape behaviour. Simple averaging to an effective event may miss that some events are more important in controlling erosion and sediment transport than others (Nichols, 2006). Furthermore, modification of the landscape by continued operation of smaller events may be (more) important in controlling morphological change (Goodrich et al., 2008; Sambrook-Smith et al., 2010).

The issue of what scales to consider and therefore what processes to resolve explicitly in a given model structure points towards a fundamental issue for geomorphologists: given natural systems often display non-linear, threshold responses, it is uncertain - and debated in the literature - to what extent small scale processes (in both space and time) control larger scale system behaviour (Lane and Richards, 1997). It is therefore uncertain to what extent fine scale processes need to be resolved explicitly in geomorphic models, or whether simpler treatments, relying on for example regime theory, are applicable (Nicholas and Quine, 2007). Choosing a specific process representation therefore reflects a specific modelling hypothesis regarding the relevant processes governing a different problem. Multiple representations, and therefore hypotheses of the same processes may require investigation (Krueger et al., 2009). To help overcome this issue, data are required to constrain model parameters and evaluate model hypothesis (Kleinhans et al., 2012).

**Data and model evaluation**

Data availability is an essential factor governing model development, as data provides the modeller with the ability to constrain model parameters and evaluate the quality of model predictions. The evaluation of model process representation is an essential step as it often occurs prior to application of models to investigate so called “what if” questions (Michaelides and Wilson, 2007; Nicholas and Quine, 2010). Such model application is often at space and time scales over which data are insufficient to differentiate between competing model representations.

![Figure 5](image-url).

*Figure 5. Simulated channel belt and floodplain evolution (e.g. by bifurcation, avulsion and aggradation) in response to base level rise (Karssenberg and Bridge, 2008).*
hypotheses (Pelletier, 2011).

In the ideal case, all process parameterisations of equations 2-4 will be known and data will be available at the scale of the model cell size to constrain model parameters and the initial model conditions. For example in soil erosion modelling distributed information on particle size may be required to parameterise both roughness for overland flow modelling and the supply limiting factors controlling sediment entrainment (Wainwright et al., 2008b). However, often such data are unavailable, or inconsistent with the scale of model application (Brazier et al., 2011).

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As a consequence, models may be calibrated by comparing model outputs to observations. In the case of catchment modelling, models are typically calibrated by adjusting internal model parameters to derive the best fit between model outputs (e.g. sediment/water flux at a catchment outlet) and the equivalent observations at specific locations (Canfield and Goodrich, 2006; Nearing, 2000). In morphological modelling, distributed model predictions may be compared to: observations or morphological change (Figure 6; Nicholas and Quine, 2007a); to results derived from models with a stronger physical basis for prediction (Nicholas, 2009); and also to results derived from physical model experiments (Nicholas et al., 2009).

A number of implicit assumptions made in model calibration may potentially undermine model application; First, it is often assumed that parameter uncertainty is the only form of modelling uncertainty; Second, that the model is equal to reality; Third, the initial states are the true initial states (e.g. the DEM is error-free); and Fourth, that the input/driving conditions and output data to evaluate model performance are true. In most, if not all situations these conditions do not hold due to the problems of modelling an open system, where true model validation and verification is impossible (Oreskes et al., 1994).

As a result of these assumptions incorrect model parameters can be identified that reproduce catchment outlet conditions (i.e. larger scale measurements) with insufficient consideration of how well they reproduce the internal spatial patterns of process-form interaction that ultimately control larger (and longer) scale response. Furthermore, model parameters may be identified that are highly unique to specific settings (Nearing, 1999; Nearing, 2000), and inapplicable elsewhere because of the non-linear open nature of natural systems. Worse still, a number of parameter combinations within a specific model may provide equally good predictions - a form of model equifinality (Brazier et al., 2000). Similarly, another form of model equifinality may occur if the data are insufficient to differentiate between competing models. Equifinality has arisen at a range of scales, from using metrics of landscape form to differentiate between transport and supply limited models of landscape evolution (Pelletier, 2011), evaluating alluvial fan evolution (Nicholas and Quine, 2010), and at smaller scales when applying models with complex, and ill constrained parameters (Brazier et al., 2000). However, equifinality is not all bad if it avoids over confidence in the information content of data and therefore the potential rejection of good model structures. In the face of equifinality, simpler models may be preferred that are justified by the data: A model is only as good as the data available to constrain model structure, parameters, and therefore predictions.
Accounting for uncertainty when developing numerical models

The preceding sections have discussed the main issues to consider in developing and applying a numerical model to address a geomorphic problem. Even if we consider our conceptual model of the system and the dominant processes to be accurate, computational resources and data availability will limit our ability to apply the preferred model at the desired scale. Furthermore, uncertainty surrounding the relationship between process dominance and scale is a fundamental geomorphic question governing model development. Therefore there is no single answer to the three questions posed in the introduction, and nor can each be answered independently. As a result of these factors a model can and will only remain as a (working) hypotheses of how processes and landforms interact. In order to develop, evaluate and use models in geomorphic enquiry, uncertainty in both data and models needs to be dealt with in a robust manner.

Considering numerical modelling from a Bayesian perspective provides a suitable framework for robust model development of non-linear open systems. As a result of modelling uncertainties we should be interested in obtaining the probability $P(\cdot)$ that the model $(M)$ and its parameters $(\theta)$ are a correct representation of reality, given the available data $(Y)$, initial model conditions $(X_0)$, and driving conditions $(D)$. In order to do this we combine our prior beliefs about the model structure, $P(M)$ and associated parameters, $P(\theta|M)$ (which are dependent on the specific model structure) with some data using a Likelihood Function (e.g. a measure of model performance based on a given dataset) $P(Y|\theta, M, X_0, D)$, to obtain our posterior belief from Bayes’ equation (Draper, 1995):

$$P(M, \theta|Y, X_0, D) \propto P(Y|\theta, M, X_0, D)P(\theta|M)P(M) \quad (5)$$

Thus, our confidence in the model is specifically dependent on the data to constrain initial conditions, driving conditions and that used to evaluate model performance.

In many applications only a single model structure is considered, and associated parameters are either derived from previous studies or are optimised to the specific data available. As a result the final two terms in Equation 5 collapse to a single set of structural assumptions. In such cases overconfidence in the data for the reasons discussed above – both in its accuracy and its general applicability to a wide range of settings – may lead to inappropriate model rejection and narrowing of the posterior probability of all possible models. This may lead to an entrenchment of modelling concepts that may prevent wider exploration of $P(M)$ and therefore potentially more appropriate models for particular circumstances (Nicholas and Quine, 2007a; Wainwright et al., 2008a).

A better position to develop models and therefore understanding of process-form interactions is to consider the uncertainty in different model parameters, and therefore consider a range of possible combinations of parameters $P(\theta|M)$ that may reproduce the data, according to a likelihood function that considers potential uncertainty in the data. Significant advances have been made in developing appropriate likelihood functions in the related discipline of hydrology that consider different forms of model uncertainty (Beven, 2006; Schoups and Vrugt, 2010). Such statistical treatments of model uncertainty require further adaptation to geomorphic problems, including potential uncertainty in model boundary conditions and elevation data (Hutton and Brazier, 2012; Nicholas and Quine, 2010; Wheaton et al., 2010). Calibration and the related sensitivity analysis conducted by exploring adequately different parameter combinations that constitute $P(\theta|M)$ can guide the modeller as to which parameters are most important in controlling system response (Hutton et al., 2012; Saltelli, 1999). Such information can be then used to guide further data collection targeted at constraining the most important parameters.

Furthermore, when different models $P(M)$ and therefore different hypotheses of form-process interactions are confronted with the same observations (Hancock et al., 2011; Krueger et al., 2009; Pelletier, 2011; Tatard et al., 2008) the strengths and weaknesses of different model structures may be identified. Such information can be used to guide further
data collection and understand the conditions (e.g. \(Y, X_0, D\)) under which different process parameterisations (e.g. \(M, \theta\)) are valid.

Therefore posterior understanding derived from comparison to data can guide further data collection (Figure 7). Depending on the similarities between the conditions used to derive the posterior (e.g. \(Y, X_0, D\)) and the newly collected data, the posterior in equation 5 (e.g. the left hand side) can then become the prior (e.g. move to the right hand side of Equation 5) for comparison to the newly collected data when combined with a likelihood function. When using model structures and parameters derived from previous applications it is up to the modeller to consider how appropriate such models are to the situation under current consideration, and whether such model application is supported by available data.

Bayes’ equation therefore provides both a conceptual and probabilistic framework for model development that can explicitly consider uncertainty in models and data, and therefore appropriately frame the use of models in developing understanding of process-form interactions. Process-form understanding is developed iteratively through continuous dialogue between models and data. Furthermore such development provides a more robust grounding for subsequent model application to investigate “what if” type questions by considering a range of possible model structures supported by available data, which will prevent overconfidence in the results of a single model prediction.

**Conclusion**

Numerical Models have, especially over recent years, become a central tool in geomorphic enquiry, and have allowed exploration of a range of system dynamics at a range of spatial and temporal scales. Appropriate use of numerical models should consider the scale of model application, the potential processes controlling landscape form at the scale of application, computational resources, data availability, and the validity of modelling concepts derived from previous modelling applications.

Given the many uncertainties governing model application, not least uncertainty regarding fundamental issues concerning process dominance and scale, an approach to model development considering such uncertainties from a Bayesian perspective is recommended. Such an approach provides a robust framework for model development, model rejection and therefore hypothesis testing that considers uncertainty in both data and models. Advances in data collection and specification of errors in available data will facilitate robust model development.

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